Homework 2

COMP221 Spring 2025 - Suhas Arehalli

Complete the problems below. Note that point values are roughly correlated with effort, but inversely correlated with expected difficulty. Check the course website & syllabus for further instructions.

If any problem is unclear, or you think you found a typo, please let me know ASAP so I can clarify!

Problems

1. **Another Quadratic Sort** *(15pts)*: Consider the sorting algorithm in Alg. 1 called SelectionSort:

Algorithm 1 Selection Sort and the Select helper function.

```
function SelectionSort(Array A)
    Let N be the length of A
    for i ← 1 to N − 1 do
        idx ←Select(A, i)
        Swap(A[idx], A[i])
    end for
end function
function Select(Array A, Integer i)
    Let N be the length of A
    idx ← i
    for j ← i + 1 to N do
        if A[idx] > A[j] then
            idx ← j
        end if
    end for
    return idx
end function
```

Prove its correctness in the following steps:

(a) *(0pts)* Run SelectionSort on an example array. Before moving on, ensure you (1) are convinced the algorithm works, (2) understand what the Select function does, and (3) intuitively understand what each iteration of each loop is doing and how it gets us toward a sorted array.

(b) *(2pts)* Provide a loop invariant that will help us prove the correctness of SelectionSort.

(c) *(2pts)* Provide a loop invariant that will help us prove the correctness of Select (and therefore help us prove the correctness of SelectionSort).

(d) *(5pts)* Prove the loop invariant you provided in part c is correct using induction. Conclude that Select is correct, for some definition of correct.

HINT: Consider the inputs of Select. *What must be true of the return value given A and i?*

(e) *(5pts)* Prove the loop invariant you provided in part b is correct using induction. Use the correctness of Select from problem c to help you.

(f) *(1pt)* Conclude that the array A is sorted using you loop invariant from part b.

2. **Recursive Linear Search** (*15pts*)

(a) Write pseudocode for a recursive linear search algorithm called RecursiveLinearSearch. This algorithm should take in an array A and some element e and return either an index i or $NULL$. If it returns an index i, the algorithm is correct if $A[i] == e$. If it returns $NULL$, $e \notin A$. (*5pts*)

(b) Prove the correctness of this algorithm, using the definition of correctness for search we discussed in class (and is summarized in part a). (*10pts*)